ANALYSIS OF NON-CLASSICALLY DAMPED STRUCTURES, METHODOLOGY AND PRACTICAL RESULTS

John D. Stevenson,
Stevenson and Associates - Europe
Alexei M. Berkovski **Oleg B. Kireev**
Victor V. Kostarev
Stevenson and Associates, Russian Office, Box 426, 197348, Saint-Petersburg, Russia, Email: cvs@cvs.spb.su

ABSTRACT

The paper presents the analytical method and main approaches that have been used for the four benchmark problems developed by BNL in the scope of the "Benchmark Program for the Evaluation of Methods to Analyze Non-Classically Damped Coupled Systems". The full investigation [1] contains the detailed description of implemented Analytical Method, solution for each of the benchmark problem with comments and detailed output results. In general, three methods have been used to resolve the problem. Two Response Spectrum Methods (RSM-I and RSM-II) can be easily implemented in engineering practice from point of view of traditionally available information for purposes of analysis. The complex-mode Time History Analysis has been utilized also to show possible deviation versus applicable engineering efforts.

METHOD OF ANALYSIS.

The equation of motion for coupled "primary-secondary" system may be expressed in the global system coordinates as:

$$\begin{bmatrix} Ks & Ksp \\ Kps & Kp+K^S p \end{bmatrix} * \begin{Bmatrix} u_S \\ u_P \end{Bmatrix} + \begin{bmatrix} Cs & Csp \\ Cps & Cp+C^S p \end{bmatrix} * \begin{Bmatrix} \dot{u}_S \\ \dot{u}_P \end{Bmatrix} + \begin{bmatrix} Ms & 0 \\ 0 & Mp \end{bmatrix} * \begin{Bmatrix} \ddot{u}_S \\ \ddot{u}_P \end{Bmatrix} =$$
$$= -\begin{bmatrix} Ms & 0 \\ 0 & Mp \end{bmatrix} * \begin{Bmatrix} Ugs \\ Ugp \end{Bmatrix} * a_g(t) \quad (1)$$

where:

Ks, Cs, Ms - stiffness, damping and mass matrix for fixed in constrained points secondary system ;

Kp, Cp, Mp - stiffness, damping and mass matrix for primary system;

Ksp, Kps, Cps, Csp - components of stiffness and damping matrix for coupled system;

$K^S p, C^S p$ - components of stiffness and damping matrix for the constraining of the secondary system to the primary system;

$a_g(t)$ - earthquake ground acceleration;

$u_S, \dot{u}_S, \ddot{u}_S$ - displacements, velocities and accelerations of secondary system;

$u_P, \dot{u}_P, \ddot{u}_P$ - displacements, velocities and accelerations of primary system;

Ugs, Ugp - influence vectors that coupled the ground motion to the corresponding degrees of freedom ;

The motion of the secondary system relatively to the primary may be expressed with aid of the following transformation:

$$u_S = \overline{u}_S + Usp * u_P \quad (2)$$

where:

$\overline{u}_S$ - "relative" displacements of the secondary system ;

Usp - transformation matrix between "constraint" and "inner" points of secondary system;

It has to be noted, that Usp may be expressed in the following form:

$$Usp = -K^{-1}s * Ksp \quad (3)$$

Based on equations (2) and (3) it is possible to transform the initial system coordinates to the new "relative" coordinates with the following transformation:

$$\begin{Bmatrix} u_S \\ u_P \end{Bmatrix} = \begin{bmatrix} I & Usp \\ 0 & I \end{bmatrix} * \begin{Bmatrix} \bar{u}_S \\ u_P \end{Bmatrix} = T * \begin{Bmatrix} \bar{u}_S \\ u_P \end{Bmatrix} \quad (4)$$

where: I - Unit Matrix, T - transformation matrix defined by (4).

The assumption of negligible character of dissipation forces for "rigid body" and "constraint" modes may be expressed as the following equations:

$$\begin{bmatrix} Cs & Csp \\ Cps & C^S p \end{bmatrix} * \begin{Bmatrix} Usp * \dot{u}_C \\ \dot{u}_C \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (5)$$

and expanding (5):

$$Csp = -Cs * Usp, \quad C^S p = U^T sp * Cs * Usp \quad (5a)$$

Substituting (4) and (5a) into Equation (1) and multiplying both it parts by matrix T^T, after corresponding transformations with accounting of (3), the equation of motion may be readily rewritten in the following form:

$$\begin{bmatrix} Ks & 0 \\ 0 & Kp + \left[K^S p - Kps * K^{-1}s * Ksp\right] \end{bmatrix} * \begin{Bmatrix} \bar{u}_S \\ u_P \end{Bmatrix} + \begin{bmatrix} Cs & 0 \\ 0 & Cp \end{bmatrix} * \begin{Bmatrix} \dot{\bar{u}}_S \\ \dot{u}_P \end{Bmatrix} +$$

$$+ \begin{bmatrix} Ms & Ms * Usp \\ U^T sp * Ms & Mp + U^T sp * Ms * Usp \end{bmatrix} * \begin{Bmatrix} \ddot{\bar{u}}_S \\ \ddot{u}_P \end{Bmatrix} =$$

$$= -\begin{Bmatrix} Ms * Usg \\ U^T sp * Ms * Usg + Mp * Upg \end{Bmatrix} * A_g(t) \quad (6)$$

Given the following modal transformation:

$$\begin{Bmatrix} \bar{u}_S \\ u_P \end{Bmatrix} = \begin{bmatrix} \varphi_S & 0 \\ 0 & \varphi_P \end{bmatrix} * \begin{Bmatrix} Xs \\ Xp \end{Bmatrix} \quad (7)$$

Equation (6) may be presented in modal terms:

$$\begin{bmatrix} \Omega_S^2 & 0 \\ 0 & \Omega_P^2 + \varphi_P^T \widetilde{K}_P^S \varphi_p \end{bmatrix} * \begin{Bmatrix} Xs \\ Xp \end{Bmatrix} + \begin{bmatrix} 2\xi_S \Omega_S & 0 \\ 0 & 2\xi_P \Omega_P \end{bmatrix} * \begin{Bmatrix} \dot{X}s \\ \dot{X}p \end{Bmatrix} +$$

$$+ \begin{bmatrix} I & \varphi_S^T Ms U_{SP} \varphi_P \\ \varphi_P^T U_{SP}^T Ms \varphi_S & I + \varphi_P^T \widetilde{M}s \varphi_p \end{bmatrix} * \begin{Bmatrix} \ddot{X}s \\ \ddot{X}p \end{Bmatrix} =$$

$$= -\begin{Bmatrix} \varphi_S^T Ms Usg \\ \varphi_P^T U_{SP}^T Ms * Usg + \varphi_P^T Mp Upg \end{Bmatrix} * A_g(t) \quad (8)$$

where:

$$\widetilde{K}_P^S = K^S p - Kps * K^{-1}s * Ksp; \quad \widetilde{M}s = U^T sp * Ms * Usp \quad (9)$$

As it was shown, Equation (8) is the modal transformation of initial Equation (1). Stiffness and Mass matrixes in this equation are partially populated, but advantage of this equation is the diagonal damping matrix, which reflects the modal dissipation for isolated primary and secondary systems.

From the other hand, it is possible to rewrite (1) in alternate form:

$$K * U + C * \dot{U} + M * \ddot{U} = F(t) \quad (10),$$

where: $U, \dot{U}, \ddot{U}$ - corresponding kinematic parameters of coupled system, K, M, C - matrixes of stiffness, mass and damping, $F(t)$ - right-hand vector of external forces. After modal decomposition of (10) we have:

$$\left[\Omega^2\right] * Y + \left[\Psi^T * C * \Psi\right] * \dot{Y} + \ddot{Y} = \Psi^T * F(t) \quad (11)$$

where: Ω, Ψ - eigenfrequencies and eigenvectores of coupled undamped system, Y - corresponding modal coordinates, which can be expressed in form:

$$U = \Psi * Y \quad (12)$$

At the same time, vector U may be expressed in terms of modal coordinates corresponding to Equation (8):

$$U = T \bmod * X = \begin{bmatrix} \varphi_S & Usp * \varphi_P \\ 0 & \varphi_P \end{bmatrix} * X \quad (13)$$

From (12) and (13) follows:

$$X = T^{-1} \bmod * \Psi * Y \quad (14)$$

After substituting X into Equation (8) and performing needed transformation the modal damping matrix corresponded to (11) may be obtained as:

$$C\,\text{mod} = \Psi^T * C * \Psi = \Psi^T * T^{-T}\begin{bmatrix} 2\xi_S\Omega_S & 0 \\ 0 & 2\xi_P\Omega_P \end{bmatrix} * T^{-1} * \Psi \qquad (15)$$

and in the final form:

$$C\,\text{mod} = \begin{bmatrix} A & C \\ B & D \end{bmatrix}$$

$$A = Ms * \varphi_S * [2\xi_S\Omega_S] * \varphi_S^{\ T} * Ms$$
$$B = -Ip * Usp^T * A \qquad (16)$$
$$C = B^T = -Ms * \varphi_S * [2\xi_S\Omega_S] * \varphi_S^{\ T} * Ms * Usp * Ip$$
$$D = Mp * \varphi_P * [2\xi_P\Omega_P] * \varphi_P^{\ T} * Mp + Ip * Usp^T * A * Usp * Ip$$

Thus, the initial task leads to solution of modal Equation (11) with the diagonal Mass and Stiffness Matrix and fully populated Damping Matrix. Evidently, there are several ways to resolve this Equation. Among them are:

- numerical time-history integration. When number of accounted modes is equal to the number of DOF, this method provides the "exact" solution.
- complex eigenvalues analysis and implementation on the basis of this solution one of Response Spectrum Methods.

The equation (11) leads to the following complex 2Nx2N-eigenvalued problem:

$$\begin{bmatrix} 0 & I \\ -\Omega^2 & -\widetilde{C} \end{bmatrix} * \begin{Bmatrix} \Psi c_i \\ \lambda * \Psi c_i \end{Bmatrix} = \lambda * \begin{Bmatrix} \Psi c_i \\ \lambda * \Psi c_i \end{Bmatrix} \qquad (17)$$

from where:

$\omega_i = |\lambda_i|$ - natural frequencies of considered system; $\xi_i = -\text{Re}(\lambda_i)/|\lambda_i|$ - damping ratios.

According to [1] the response of considered system may be obtained in the following form:

$$u(t) = \sum \left\{ u_i^d * x_i(t) + u_i^v * \dot{x}_i(t) \right\} \qquad (18)$$

where:

$$u_i^d = \Phi * \Psi c_i^d \qquad \Psi c_i^d = -2\,\text{Re}(\overline{\lambda}_i * f_i * \Psi c_i)$$
$$u_i^v = \Phi * \Psi c_i^v \qquad \Psi c_i^v = -2\,\text{Re}(f_i * \Psi c_i) \qquad (19)$$
$$f_i = \frac{\lambda_i * \Psi^T c_i * F}{\Psi^T c_i * (\lambda_i^2 * I - \Omega^2) * \Psi c_i}$$

the functions $x(t), \dot{x}(t)$ may be readily founded from the 1 DOF equation:

$$\ddot{x}_i + 2\xi_i\omega_i\dot{x}_i + \omega_i^2 x_i = -\ddot{u}_g(t) \qquad (20)$$

So, any response of considered system then may be obtained from:

$$R(t) = \sum \left\{ R_i^d * x_i(t) + R_i^v * \dot{x}_i(t) \right\} \qquad (21)$$

$$R_i^d = q^T * u_i^d; \qquad R_i^v = q^T * u_i^v$$

For Response Spectrum Method the expressions (21) may be reformulated to double sum combination:

$$R^2 = \sum_i \sum_j \left\{ \begin{matrix} R_i^d R_j^d \varepsilon_{ij}^d S_d(\omega_i\xi_i) S_d(\omega_j\xi_j) + R_i^v R_j^v \varepsilon_{ij}^v S_v(\omega_i\xi_i) S_v(\omega_j\xi_j) + \\ + \quad 2 R_i^d R_j^v \mu_{ij}^d S_d(\omega_i\xi_i) S_v(\omega_j\xi_j) \end{matrix} \right\} \qquad (22)$$

where:

$$S_d(\omega_i\xi_i) = \max|x_i(t)|$$
$$S_v(\omega_i\xi_i) = \max|\dot{x}_i(t)| \qquad (23)$$

$\varepsilon_{ij}^d, \varepsilon_{ij}^v, \mu_{ij}$ - correletion coefficients

The values of $\varepsilon_{ij}^d, \varepsilon_{ij}^v, \mu_{ij}, S_d(\omega_i\xi_i), S_v(\omega_i\xi_i)$ may be obtained directly from the Time History Acceleration according to the following formulas:

$$\varepsilon_{ij}^d = \frac{\int x_i(t) x_j(t) dt}{\sqrt{\int x_i^2(t)dt * \int x_j^2(t)dt}}$$
$$\varepsilon_{ij}^v = \frac{\int \dot{x}_i(t) \dot{x}_j(t) dt}{\sqrt{\int \dot{x}_i^2(t)dt * \int \dot{x}_j^2(t)dt}} \qquad (24)$$
$$\mu_{ij} = \frac{\int x_i(t) \dot{x}_j(t) dt}{\sqrt{\int x_i^2(t)dt * \int \dot{x}_j^2(t)dt}}$$

This approach in the current evaluation has been realized for the considered benchmark problems (in the further consideration - RSM-I). Since the values of all cross-correlation coefficients are calculated here practically from the "exact" solution, such procedure may show the realistic limits of Response Spectrum Method versus Time History Analysis. But for engineering purposes for calculation of these cross-correlation coefficients the following expressions may be used [2]:

$$\varepsilon_{ij}^{d} = \varepsilon_{ij}^{v} = \frac{8\sqrt{\xi_i \xi_j}\,(\xi_i + r\xi_j)r^{\frac{3}{2}}}{(1-r^2)^2 + 4\xi_i\xi_j r(1+r^2)^2 + 4(\xi_i^2+\xi_j^2)r^2}$$

$$\mu_{ij} = 0 \qquad (25)$$

$$S_v(\omega_i,\xi_i) = \omega_i * S_d(\omega_i,\xi_i)$$

where:

$$r = \frac{\omega_j}{\omega_i} \qquad (26)$$

Namely for the last expressions the alternate simplified CQC Response Spectrum Method (RSM-II) has been developed on the basis of technique described in [3]. The accounting of high frequency modes has been performed according to [2].

The figure 1 shows the principal flow chart for described above methodology.

The described above analytical procedure has been implemented in modified for these purposes computer program dPIPE ([4], [5]).

RESULTS OF ANALYSES AND CONCLUSIONS.

The full set of analysis results has been presented in report [1]. Every set of solutions contains results for complex-mode Time History Analysis and also for two described above variants of Response Spectrum Method (RSM-I and RSM-II). Additional information (e.g. eigenfrequencies for primary, secondary and coupled undamped systems, modal damping end frequencies for complex modes, etc.) is given in corresponding tables. All Time History Analyses have been performed with integration step DEL=0.004 sec. For RSM-II the value of Cut-of-Frequency was defined as 30 Hz.

On the basis of obtained results the following main conclusions may be stated:

1. Proposed Response Spectrum Method (RSM-II) can be easily implemented in engineering practice from point of view of traditionally available information for purposes of analysis:
- modal properties of primary system or its simplified (stick) model ;
- full information for secondary system;
- set of Response Spectra for different damping ratios.
2. Preliminary comparison of RSM results versus THA results have shown that RSM-I method leads to more accurate solution. It means that additional improvement of proposed method may be obtained using more sophisticated procedure for cross-correlation coefficient calculation.
3. During actual investigation it was founded that for primary-secondary systems with closely spaced frequencies use of the absolute sign in front of the double summation recommended US NRC shows an essential error even for classically damped systems.
4. The simplified analysis of the Equation (11) with diagonal components of damping matrix only, which is often used in engineering practice, has been carried out additionally. This analysis was performed for the Problem #4. Obviously, that this method leads to essential errors not only versus accurate solution, but also versus other proposed methods and can't be recommended.

On the basis of obtained results it seems to be necessary and very important to investigate an extra case: the phenomena of local damping. Such a phenomena exists and widely dispersed due to numerous implementations of viscous dampers for vibration and seismic protection of structures, systems and piping. In dynamic analysis taking into account the simplified mathematical models of such devices (either elastic or ideal viscous part) can dramatically change the actual dynamic response ([6] – [9]).

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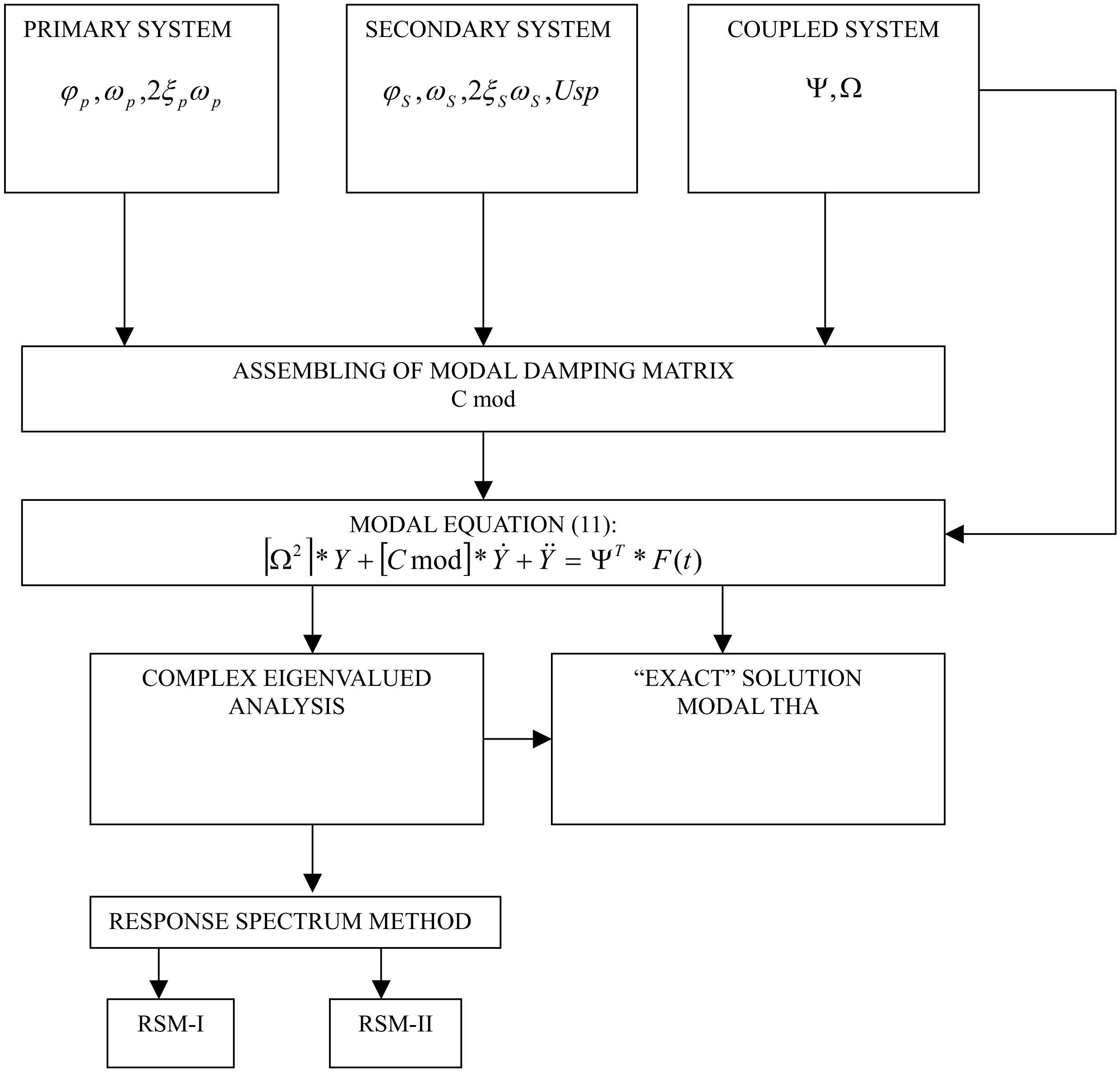

Figure 1. Principal Diagram for Proposed Analytical Method