Transactions, SMiRT-22
San Francisco, California, USA - August 18-23, 2013
Division III

GENERATION OF AN ARTIFICIAL TIME HISTORY MATCHING MULTIPLE-DAMPING FLOOR DESIGN RESPONSE SPECTRA

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ABSTRACT

The method of generation of an artificial time history matching multiple-damping response spectra is proposed. The synthesis algorithm based on spectral analysis and multiparametric optimization is adduced. The strong points of developed method in comparison with methods that operate with only one damping response spectrum are discussed through the example of seismic analysis of feed water pipeline.

INTRODUCTION

Modern approaches to the seismic design of nuclear power facilities mean that in addition to intensity that characterizes level of seismic hazard zone of construction design response spectra (RS) should be taken in consideration.

Time histories matching multiple-damping design response spectra are required for nonlinear analytical models, for structures that have different damping at their parts and for structures with a local damping such as piping protected by viscous dampers. This requirement is given in USNRC (2007): “In practical seismic analysis <…> damping is also used to account for many nonlinear effects such as changes in boundary conditions, joint slippage, concrete cracking, gaps, and other effects that tend to alter response amplitude. In real structures, it is often impossible to separate "true" material damping from system damping, which is the measure of the energy dissipation, from the nonlinear effects.”

Most of the existing methods of synthesis of time histories are operated with only one damping response spectrum. Let us demonstrate the example of such approaches.

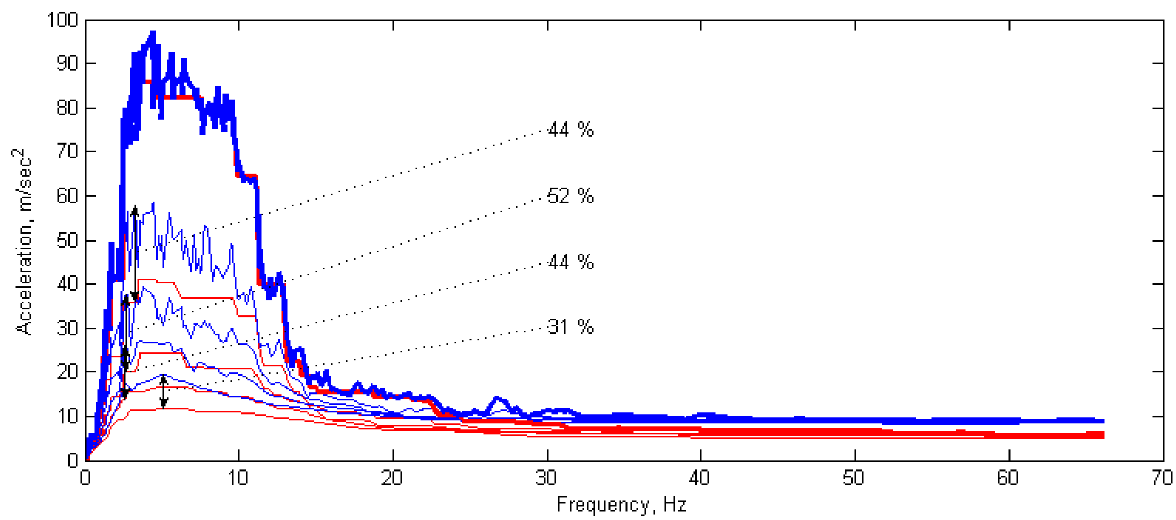

Figure 1. Blue lines – set of response spectra matching time-history generated for 0.5 % damping response spectrum, red lines – target set, thick lines – 0.5 % damping response spectra.

Figures 1 – 3 contain the set of design response spectra (red lines) defined for such damping ratios as 0.5, 2, 5, 10 and 20 %, which will be called further as the target set. The time histories matching 0.5, 5 and 20 % spectra was generated and response spectra calculated from these time histories were drawn with thick blue lines in Figures 1 – 3 respectively. Thin blue lines in these figures correspond to calculated spectra for the rest damping ratios.

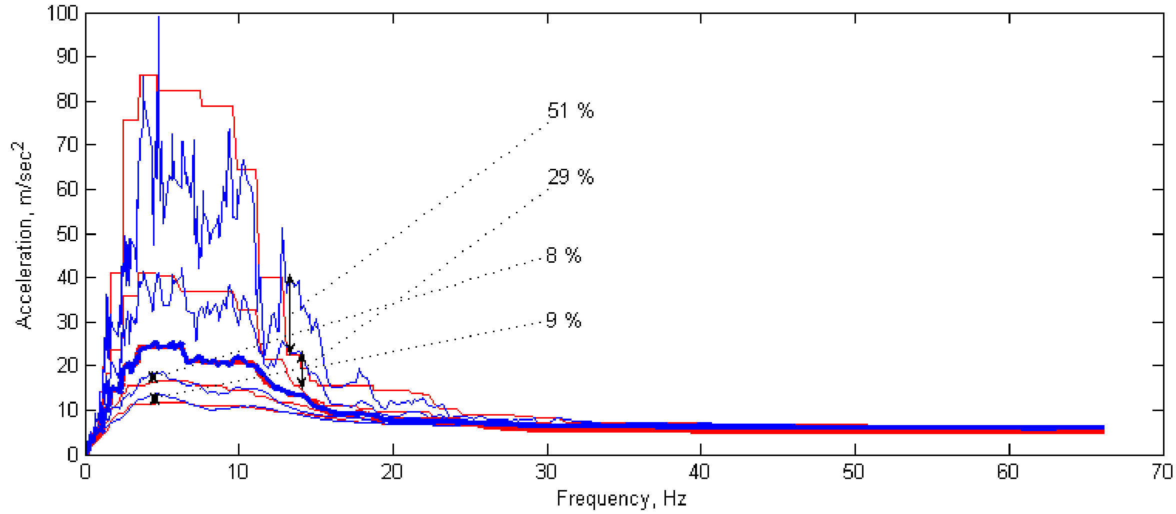

Figure 2. Blue lines – set of response spectra matching time-history generated for 5 % damping response spectrum, red lines – target set, thick lines – 5 % damping response spectra.

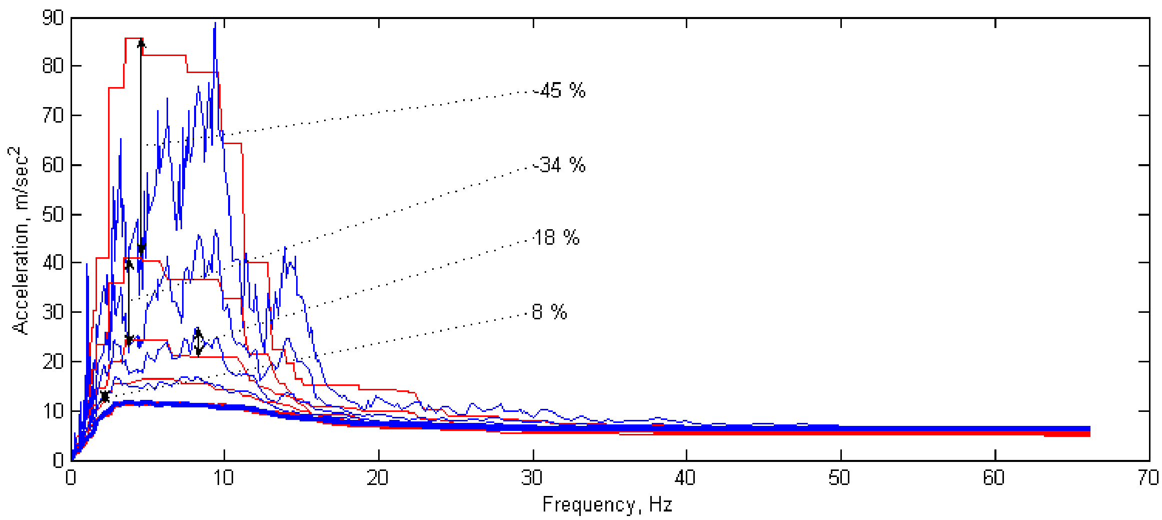

Figure 3. Blue lines – set of response spectra matching time-history generated for 20 % damping response spectrum, red lines – target set, thick lines – 20 % damping response spectra.

Double arrows in Figures 1 – 3 show relative errors of calculated values in percentage terms. Obviously, methods that lead to such large mistakes, are not convenient for the analysis of structures in cases when a matching of time history to a whole set of response spectra is essential.

SOLUTION OF THE PROBLEM

It is extremely difficult problem to find a time history matching multiple-damping spectra in time domain because representation of signal in time domain doesn't give any information about frequency components. Application of Fourier transform which make it possible to get spectral representation of signal (dependence of amplitude upon frequency) is appropriate to find an artificial time history. The proposed method for the synthesis of time histories is iterative and consist of following steps:

- choice of the initial approximation;
- forming the optimization parameters' vector;
- evaluation of the goal function;
- minimization of the goal function;
- finding the solution.

Choice of the Initial Approximation

Generated time histories should meet requirements of the regulatory design guides. Some of these requirements impose restrictions on choice of the initial approximation. Thus, according to ASCE/SEI 43-05 (2005) records shall have a time increment dt of at most 0.01 s. Artificial time histories shall be numerically developed so that they reasonably represent the ground motion expected for the site. Duration enveloping function parameters depending on magnitude are presented in Table 1.

Table 1: Duration enveloping function parameters.

Magnitude	Rise time, sec (t_r)	Duration of strong motion, sec (t_m)	Decay time, sec (t_d)
7,0-7,5	2	13	9
6,5-7,0	1,5	10	7
6,0-6,5	1	7	5
5,5-6,0	1	6	4
5,0-5,5	1	5	4

The research is operated with the enveloping function f_{env} of the form

$$f_{env}(t) = \begin{cases} \sin\left(\dfrac{2\pi t}{4t_r}\right), & 0 \le t \le t_r; \\ 1, & t_r \le t \le t_r + t_m; \\ \sin\left(\dfrac{2\pi t}{4t_d}\right), & t_r + t_m \le t \le t_r + t_m + t_d. \end{cases}$$

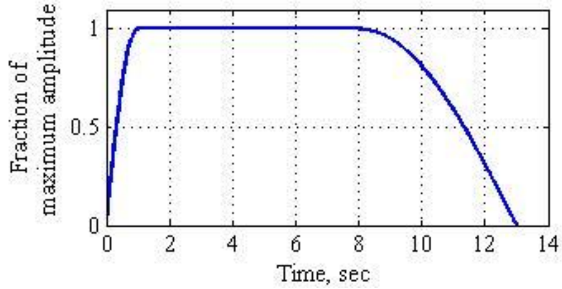

Figure 4. Enveloping function for earthquake of magnitude 6.

Knowing a time increment and enveloping function it is possible to define the number of points of desired time history as $np = (t_r + t_m + t_d)/dt$. Since the algorithms of the fast Fourier transform (FFT) run at maximal speed when number of points is equal to a power of two, let us put the number of points of the time history $N = 2^k$ so that $2^{k-1} < np \le 2^k$. Herewith formally the duration of the time history (and hence the duration of the enveloping function) increases on $(N - np)\cdot dt$ seconds. However, let us assume that $f_{env}(t) = 0$ with $t_r + t_m + t_d < t \le t_r + t_m + t_d + (N - np)\cdot dt$. This assumption allows to keep the shape of the enveloping function satisfying ASCE 4-98 (1998) and obtain the optimal number of parameters for the FFT.

Obviously, the seismic action is by nature superposition of harmonics. Let the number of harmonic components is $N/2+1$, then it is possible to form an initial approximation to the solution. Let us assume amplitudes and phase of harmonics are equal to random numbers in ranges from 0 to 1 and from 0 to 2π respectively. Further it is necessary to assemble the time history as the superposition of declared harmonics, multiple it on the enveloping function and find the maximum absolute value of received time history – $\max_t a_0(t)$. Finally corrected initial approximation to the solution can be found by multiple the amplitudes on value $\dfrac{ZPA}{\max_t a_0(t)}$, where ZPA – zero-period acceleration.

Forming the Optimization Parameters' Vector

Applying the Fourier transform to the initial approximation, we obtain $a\ (t) = \sum_{k=1}^{N} X_k e^{ik\omega t}$, where X_k – k-th complex amplitude, $k\omega$ – k-th circular frequency of the harmonic oscillation. The switch from the coefficients of the Fourier transform to the amplitudes and phases of the harmonics contained in the original signal will yield

$$A_k = \frac{1}{N}\sqrt{(\mathrm{Re}(X_k) + \mathrm{Im}(X_k))},$$

$$\varphi_k = arctg\left(\frac{\mathrm{Im}(X_k)}{\mathrm{Re}(X_k)}\right), \quad k = \overline{1, N/2}.$$

Then the optimization parameters' vector is $x = [A_1, A_2, ..., A_{N/2}, \varphi_1, \varphi_2, ..., \varphi_{N/2}]_{N\times 1}$.

Evaluation of the Goal Function

At each iteration the value of the goal function is evaluated in the parameter space as the difference between the original spectra and the spectra calculated from the current approximation to the solution:

$$f\{a(t)\} = \left(\frac{1}{J}\sum_{j=1}^{J}\frac{1}{nfreq^j}\sum_{\omega\in\Omega}(RS^j{}_{t\arg et}(\omega) - RS^j{}_{current}(\omega))^2\right)^{1/2}. \tag{1}$$

Here $RS^j{}_{t\arg et}$ – j-th original (target) response spectrum, $RS^j{}_{current}$ – the current approximation to the j-th target response spectrum, J – number of spectra, $nfreq^j$ – number of frequencies of j-th RS.

According to USNRC (2007) spectral acceleration at 5 % damping shall be computed at a minimum of 100 points per frequency decade, uniformly spaced over the log frequency scale from 0.1 Hz to 50 Hz or the Nyquist frequency. The comparison of the response spectrum obtained from the artificial ground motion time history with the target response spectrum shall be made at each frequency computed in the frequency range of interest.

In practice, the original spectra are often defined at a small amount of points. So before the goal function will be evaluated, the target response spectra should be interpolated in the new frequency range which is found by the rule $\omega_{i+1} = \omega_i (1+\zeta_j)$, where ζ_j – damping of j-th spectrum.

For computation of RS the equation of the oscillator

$$\ddot{x} + 2\zeta\dot{x} + \omega^2 x = -a(t)$$

(where $\ddot{x}, \dot{x}$ and x – acceleration, velocity and displacement correspondingly, ω – oscillator's circular frequency, ζ – damping coefficient) is numerically integrated for the entire range of frequencies and all damping ratios. Function $a(t)$ in the right part of this equation is restored by inverse Fourier transform of parameters vector at the current iteration. Additionally the current approximation to the time-history of ground motion is enveloped, i.e. multiplied by the enveloping function, and finally passed as an argument to the goal function.

Obviously, the presence of high frequencies in the Fourier spectrum leads to an unacceptable increase of the RS in the high-frequency region. To avoid this situation the Fourier coefficients corresponding to high frequencies are considered equal to zero. Let us consider the procedure of cutting high frequencies.

There is a well-known Nyquist-Kotelnikov sampling theorem (Marple (1987)), which states: if a function contains no frequencies higher than B hertz, it is completely determined by giving its ordinates at a series of points spaced 1/(2B) seconds apart. The theorem provides the way to calculate the frequencies at which the original signal is decomposed in the Fourier spectrum. Let dt – sampling interval, N – number of samples, then the Nyquist frequency is equal to $\upsilon_{Nyq} = 1/2dt$.

Since the FFT algorithm shifts a negative part of the Fourier spectrum to the right, one can construct the Fourier spectrum for $N+1$ frequencies from $-\nu_{Nyq}$ to $+\nu_{Nyq}$ in increments $\delta\nu = 1/Ndt$. In this case X_0 corresponds to the frequency $\nu = 0$, positive frequencies are in the range of numbers from 1 to $N/2$, the remaining values correspond to negative frequencies.

If cut-off frequency is set equal to ν_{cut}, it is possible to find in the range of negative frequencies such a number k that the following inequality is fairly: $\nu_k \le \nu_{cut} \le \nu_{k+1}$. Then to avoid an unacceptable increase of RS at the right of ν_{cut} it is sufficiently to let $X_i = 0$ with $i = N/2 - n_{cut} ... N/2 + n_{cut}$, where $n_{cut} = N - k$.

Figures 5, 6 show an application of the described method for $\nu_{cut} = 30$ Hz.

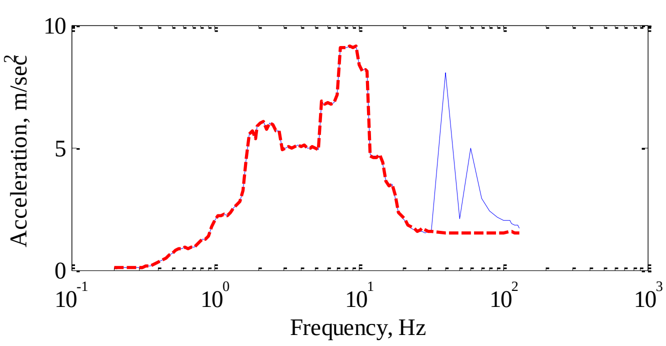

Figure 5. Procedure of cutting of high frequencies. Response spectra calculated from time histories in Figure 6 (blue – before procedure, red – after procedure).

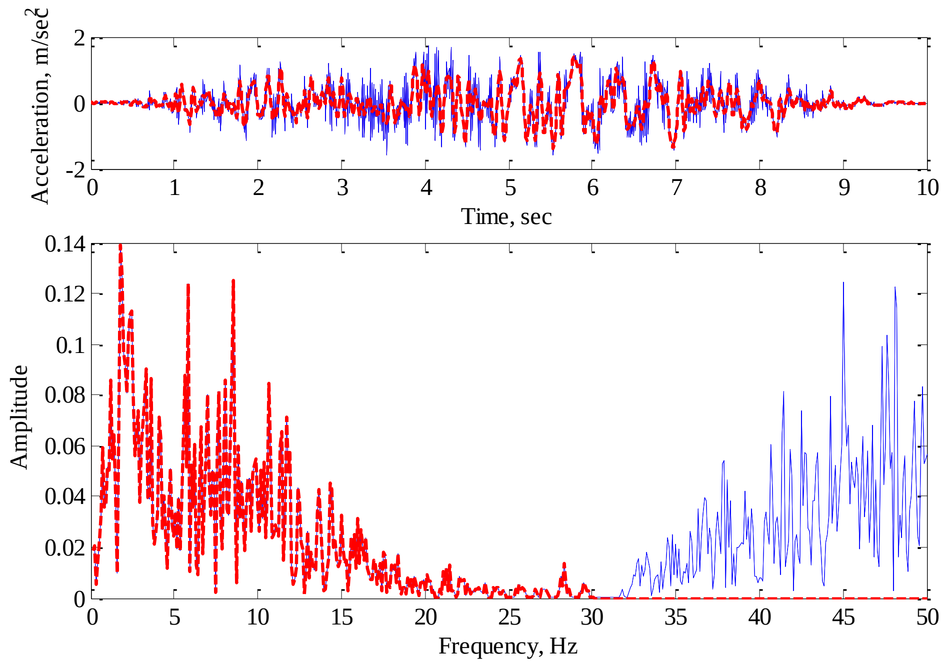

Figure 6. Procedure of cutting of high frequencies. Time histories and Fourier spectra (blue – before procedure, red – after procedure).

Minimization of the Goal function

A modified Hooke-Jeeves algorithm (Bunday (1984)) is used for minimization of goal function. The Hooke and Jeeves method for finding an optimal solution consist of two kinds of moves: an exploratory and a pattern move. The exploratory move is accomplished by doing a coordinate search in one pass through all the variables. This gives a new "base point" from which a pattern move is made. A pattern move is a jump in the pattern direction determined by subtracting the current base point from the previous base point.

Considering the procedure of cutting of high frequencies the number of optimization parameters is reduced to $N-2n_{cut}$. For instance, if duration of desired time history is 40 seconds, time increment is 0.01 sec and cut-off frequency is defined as 30 Hz, then number of points $np = 4000$, number of parameters $N = 2^{12} = 4096$, the Nyquist frequency $\nu_{Nyq} = 50$ Hz, $n_{cut} = 819$, so in this case the number of optimization parameters can be reduced from 4096 to 2458.

Let the initial base point is $x = [A_1, A_2, ..., A_{N/2-n_{cut}}, \varphi_1, \varphi_2, ..., \varphi_{N/2-n_{cut}}]_{(N-2n_{cut})\times 1}$ and increments for the variables of x are

$$step_i = \begin{cases} \dfrac{ZPA}{2\max_t a_0(t)}, & if\ i = \overline{1, N/2-n_{cut}} \\ \pi, & if\ i = \overline{N/2-n_{cut}+1, N-2n_{cut}} \end{cases}.$$

In the classical Hooke-Jeeves algorithm if the coordinate search through all variables does not improve the goal function, the exploratory move is repeated in the same base point but increments should be twice reduced. Let us modify the algorithm as follows: the increments should be divided in half if there were only few successful steps during a coordinate search (less than 20 % of total number of parameters) or $\dfrac{f_i - f_{i+1}}{f_i} < 0.05$. Step is called successful if it improves the goal function. The ratio $\dfrac{f_i - f_{i+1}}{f_i}$ shows how quickly the goal function decreases, the increments are considered inefficient for further optimization in case of falling this value lower than 0.05.

The clear distinction between the modified algorithm and the classical one is that in the modified algorithm a pattern move is repeated while the goal function improves, whereas in the classical version a pattern move is applying only once after every exploratory move.

Finding the Solution

Desired time history is a minimum point of the goal function:

$$a(t) = \arg\min\left(f\{a(t)\}\right).$$

There are a great amount of time histories matching target set of RS. In general, the ideal solution of the problem ($f\{a(t)\} = 0$) does not exist for reasonable restrictions on duration of time history. So the aim of research is to find one of the local minimum of the goal function, but not the global one. If the obtained time history meets the requirements of various regulatory guides, the problem is considered solved.

PRACTICAL APPLYING

Benefits of the proposed method of generation of an artificial time history are shown in this section through the example of seismic analysis of feed water pipeline located between steam generator and hermetic penetrant. The computer model of the above-mentioned structure was created by means of dPIPE5 (2007) software and presented in Figure 7.

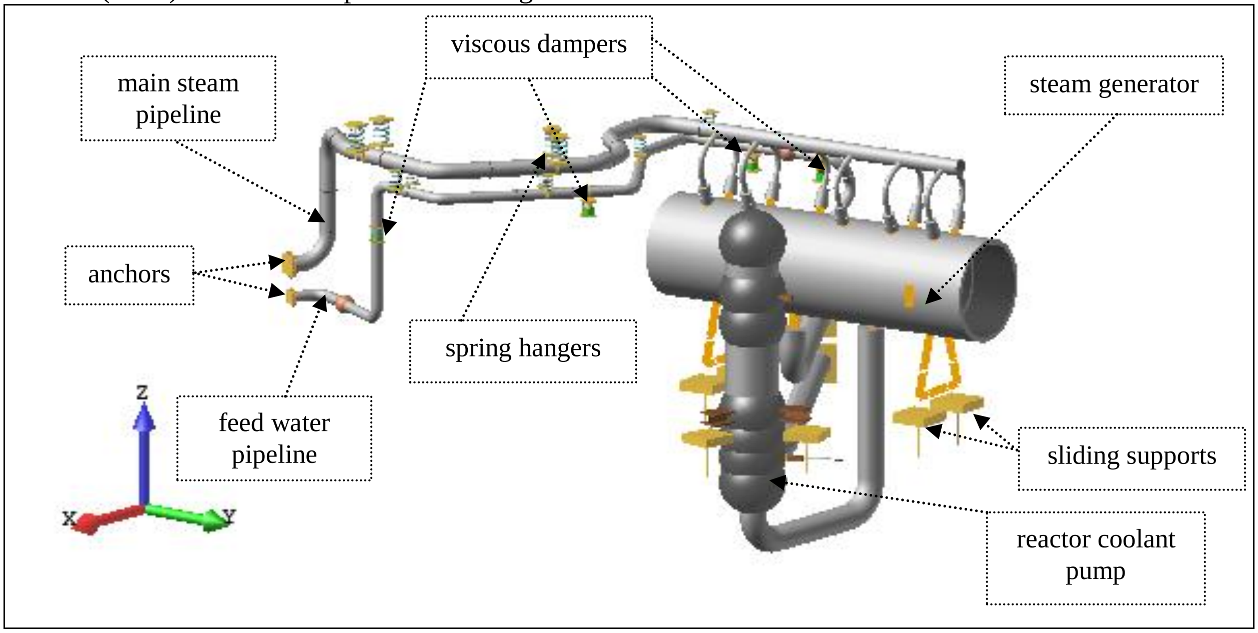

Figure 7. Pipeline model.

Seismic characteristics at the level of the pipeline were given by set of three-directional time histories. Results of analysis of the model without dampers where this set was used as an input action were considered as standards. Hereafter the three-directional set of response spectra for 2, 5 and 10 % damping was calculated from given time histories and called the target set. Then 10 sets of three-directional artificial time histories matching the target set were generated by the proposed method (let us denote it as Approach A). It should be noted that any number of time histories' sets is available due to randomness in choice of initial approximation. Another 10 sets adequately matching only 5 % damping RS were generated by methods mentioned in the introduction (let us call it as Approach B). Example of data for X-direction syntheses is presented in Figure 8, the mean percentage errors are given in parentheses. The left part of Figure 8 shows the target set of RS (red lines in the top plot) computed from the original time history (red line in the bottom plot), the time history generated by the proposed method (blue line in the bottom plot) and the calculated from the obtained time history set of RS (blue lines in the top plot). The right part of Figure 8 illustrates the time history generated during the Approach B (the bottom plot) for 5 % damping RS in the top plot; the compatibility of this time history with the rest RS in the target set is shown in the middle plot.

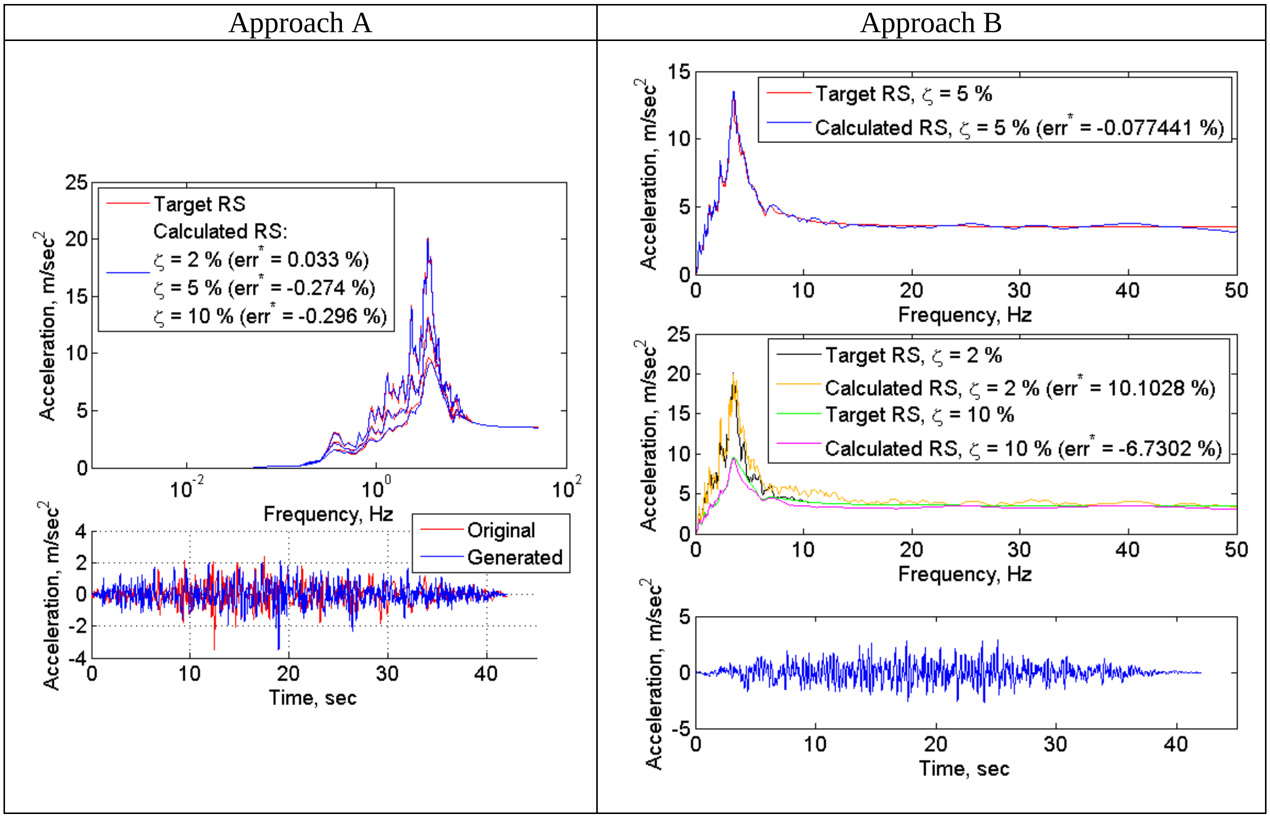

Figure 8. X-direction data: original and generated time histories, target and calculated RS.

As to the accuracy of syntheses, the time histories obtained by the Approach A are such that each goal function determined in Equation 1 is less than 0.1. Mean percentage errors of all calculated sets of spectra at linearly distributed points in range from 0.05 to 50 Hz with increments of 0.05 Hz are displayed in Tables 2, 3.

Table 2: Mean percentage errors of calculated RS sets. Approach A.

	2 %	5 %	10 %
X	0.3177	-0.2047	-0.2829
Y	-0.4186	-0.1831	-0.1476
Z	1.3672	1.7129	1.9122

Table 3: Mean percentage errors of calculated RS sets. Approach B.

	2 %	5 %	10 %
X	10.7853	1.1113	-4.7681
Y	8.0487	5.5789	4.3918
Z	8.2749	6.3849	5.5803

The output values which were compared with the standards are stresses in the 62 nodes of the feed water pipeline, loads on the 6 spring hangers and on the 7 sliding supports, 6 forces and 6 moments at the anchors, loads on the 10 snubbers, so 97 values for each of 10 sets were considered. The percentage errors of the values observed by both approaches to the standards were computed. The probability density functions of the percentage errors in comparison with the probability density functions of the normal distribution with the same mathematical expectations μ and dispersions σ^2 are pictured in Figure 9.

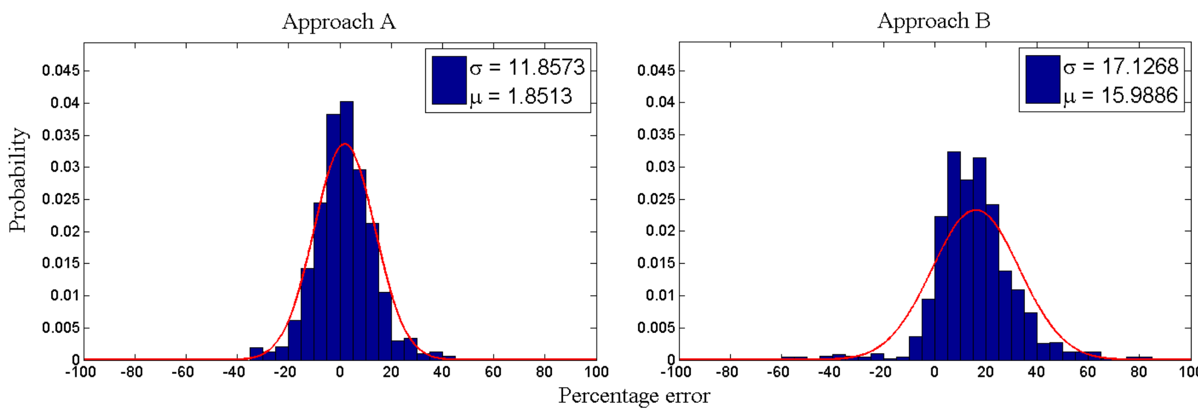

Figure 9. Probability density functions (blue bars – the percentage errors, red lines – normal distribution) in case of the original pipeline's model.

The similar research was held for the same model equipped with dampers. The obtained results are shown in Figure 10. Apparently, Approach A yields smaller stretching of percentage errors in both analyzes. Moreover, a standard deviation of the errors in case of Approach A decreased after adding of dampers, but became even larger in case of Approach B. Besides, essential displacement of a mathematical expectation from zero in a positive way in case of Approach B indicates on conservatism of the analysis.

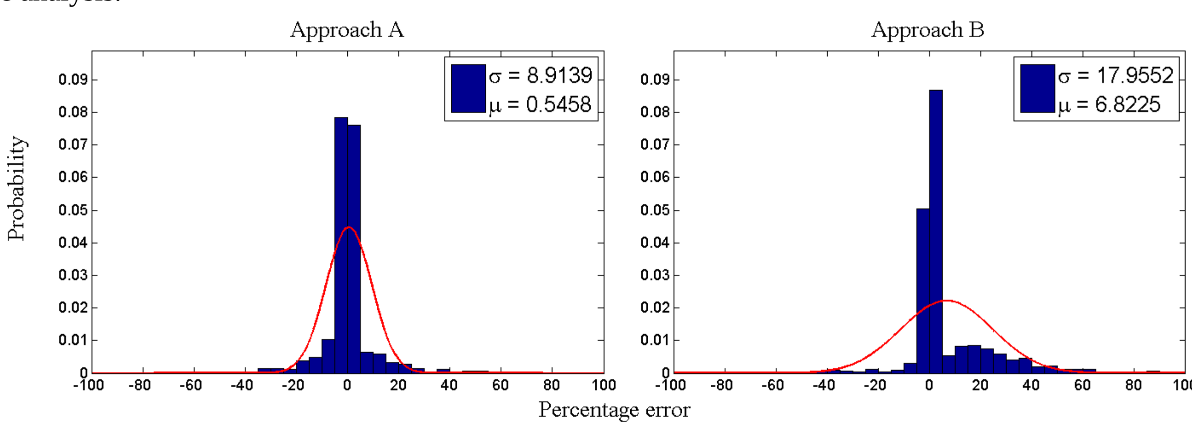

Figure 10. Probability density functions (blue bars – the percentage errors, red lines – normal distribution) in case of the pipeline's model with dampers.

CONCLUSION

The proposed synthesis algorithm allows to obtain an artificial time history compatible with the whole set of design response spectra avoiding the drawbacks of the most existing generation methods that operate with only one response spectrum. The considered example of seismic analyses of the feed water pipeline that used generated time histories demonstrates the advantages of the adduced method, especially in case of installed dampers.

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